

Mathematics, BA

Program Description

The BA Undergraduate Program in the Department of Mathematics seeks to provide a high-quality education to our students, providing both depth and breadth in various fields of Mathematics, with an emphasis on the teaching aspects of the field. The breadth of knowledge may include but is not limited to algebra, analysis, topology, number theory, differential equations, and numerical analysis.

Our mission is to prepare our students not simply to feed the workforce with qualified professionals, but to produce catalysts of change who will propel our society, our economy, and our scientific enterprise beyond our current achievements. The program is structured to foster strong analytical and quantitative reasoning, aiming to connect the highest abstract concepts with both the concrete origins of the subject and their direct applications to important societal problems. Teacher preparation is one of the most important tasks for an academic program, for it has a direct and fundamental impact in our society. We strive to produce knowledgeable teachers who will be effective professionals and capable of working in diverse environments while maintaining the highest quality of instruction.

Outcome 1 – Fundamentals of Algebraic and Geometric Methods

Students will be able to construct models for mathematical problems using algebraic methods and geometric axioms.

Measure 1.1 – Applied Algebra: Comprehensive Final Exam

Data Collection: *Applied Algebra (Math 433)* is an introduction to modern algebra with discussions on groups, rings, and fields with emphasis on modular arithmetic. The course includes applications to number theory, coding theory, and other areas.

Each of the topics listed in the description above is essential for developing mathematical models and then making decisions and predictions based on these models. The Final Exam in Math 433 covers comprehensive techniques for developing and applying these models.

Methodology or data analysis strategy: Comprehensive Final Exams in Math 433 are designed, administered, and graded by course instructors.

The associate head works with Math 433 course instructors to obtain numerical data on this measure for BS-APMS students during the reporting period.

[See Summary of M433 under *Supporting Documentation*.]

Target 1.1

Standard: Students must achieve at least 65% of the maximal numerical grade assigned to the Comprehensive Final Exam in Math 433.

- This represents the minimal level of mastery needed for a student to be successful in professional careers, subsequent courses, or graduate studies in applied mathematics.

Target: 70% of the students will meet or exceed the minimal standard.

- This target represents a goal to be attained by the students in the program and will be revised as data are collected and actions plans are implemented.

Finding 1.1: Met

For the Mathematics BA-MATH degree:

During the 22-23 academic year, the average on the Math 433 Comprehensive Final Exam was 85.9%. On this measure, 100% of the students whose major is a Bachelor of Arts in mathematics achieved the standard of a 65% or higher on the final exam. Thus, the target of 70% of the students meeting this standard was met. Attached is a summary of these results.

The attached document shows the trends in the averages on the Math 433 Comprehensive Final Exam over the last two academic years. Math 433 is a new measure for the Bachelor of Arts in mathematics program, thus we have only collected data for the 21 - 22 and 22 - 23 academic years. When we compare the average on the final exams, we see there was approximately a 10% decrease. We will monitor this measure over the next few years to determine if the decrease was anecdotal or if there is a trend we need to address.

Measure 1.2 – Modern Geometry: Comprehensive Final Exam

Data Collection: *Modern Geometry (Math 467)* covers the modern development of Euclidean geometry, within its historical and philosophical context, along with a full discussion of the independence of the parallel postulate and the development of hyperbolic non-Euclidean geometry.

The purpose of Math 467 is to acquaint students with both Euclidean and non-Euclidean geometry, with an emphasis on the distinction between reasoning from abstract axioms containing undefined terms and reasoning from facts about things the students already know something about. Each problem on the final exam in Math 467 uses the geometric methods described above.

Methodology or data analysis strategy: Comprehensive Final Exams in Math 467 are designed, administered, and graded by course instructors.

The associate head works with Math 467 course instructors to obtain numerical data on this measure for BS-APMS students during the reporting period.

[See Summary of M467 under *Supporting Documentation.*]

Target 1.2

Standard: Students must achieve at least 65% of the maximal numerical grade assigned to the Comprehensive Final Exam in Math 467.

- This represents the minimal level of mastery needed for a student to be successful in professional careers, subsequent courses, or graduate studies in applied mathematics.

Target: 70% of the students will meet or exceed the minimal standard.

- This target represents a goal to be attained by the students in the program and will be revised as data are collected and actions plans are implemented.

Finding 1.2: Met

For the Mathematics BA-MATH degree:

During the 22-23 academic year, the average on the Math 467 Comprehensive Final Exam was 77.0%. On this measure, 90.5% of the students whose major is a Bachelor of Arts in mathematics achieved the standard of a 65% or higher on the final exam. Thus, the target of 70% of the students meeting this standard was met. Attached is a summary of these results.

The attached document shows the trends in the averages on the Math 467 Comprehensive Final Exam over the last three academic years. Compared to previous years, the trend of this measure appears to be fairly steady for BA-MATH students. The small spike in the 21-22 average and percentage of students who met the target can be explained by an abnormally small sample size of BA-MATH students who completed the course during that academic year.

Use of Results

For the two measures we are reporting this cycle, Math 433 (Applied Algebra) Comprehensive Final Exams and Math 467 (Modern Geometry) Comprehensive Final Exams, both targets were met. However, there is some weakness indicating room for improvement.

Success in both of these courses, but especially Math 433, requires students to have a sound footing in Linear Algebra. Particularly there is a marked need for a facility with abstract vector spaces and their associated linear transformations. When dealing with objects defined over general rings and fields, which is essential for Math 433, it is important for students to be able to adjust their perspective between standard views of vectors in Euclidean space and their equivalent but more robust versions in abstract spaces.

For our students the primary source for this information is taught in Math 323 (Linear Algebra), but it has recently come to our attention that the level of abstraction in our Math 323 courses varies considerably from one instructor to another. Moreover, Math 323 is a bridge course between Math 300, Foundations of Mathematics, which introduces rigorous proof-writing, and most, if not all, upper-level math courses. As such it is vital that Math 323 students build on their logic and proof skills in order to utilize them fully in their upper-level course work.

Math 323 instructors often also teach Math 304, which is our service linear algebra course on vectors in Euclidean space. The changes proposed below will also serve as a way for our instructors to more clearly delineate the topics between the two courses. Indeed some instructors have mentioned that

Math 323 is “essentially the same” as Math 304, but this is not supposed to be the case, and one of our goals is to dispel this misconception.

To address these deficiencies the Math Department Undergraduate Program Committee proposes the following actions:

1. The Associate Head for Undergraduate Programs will meet with Math 323 instructors to clearly outline expectations for the course. One of the main goals will be to make sure certain topics regarding abstract vector spaces (spaces of functions, matrix representations with respect to arbitrary bases, diagonalization of linear operators, abstract inner product spaces and orthonormalization, etc.) are covered adequately. We also want to distinguish the topics in Math 323 from Math 304, which is our service linear algebra course for vectors in Euclidean space.
2. Instructors will also be urged to incorporate more logic, proof-writing, and problem solving techniques from Math 300 into Math 323. Students will not only be learning proofs of important theorems, but be required to supply their own proofs and logical arguments on homework and exams. In this way they will build solid foundations for the more abstract constructions and be better prepared for upper-level work.

It will take some time to measure the effects of this change on performance in Math 433, but we anticipate being able to report on improvements in final exam scores in the coming years. The next time we report on these measures in the 2025-26 cycle, we will be able to update on the status of these changes.

The Undergraduate Program Committee in the Department of Mathematics will work with Math 323 instructors to design and implement these changes. This committee consists of six Math Department faculty and is chaired by the Associate Head for Undergraduate Programs.

Status Update on a Previous Action

In the 2021-22 cycle we identified the need to develop a 1-credit hour course for first year APMT and MATH majors that introduced students to programming (in python) and scientific word processing (in LaTeX) so that students become well versed with the technology they will need for future course work. The overall goal was two-fold:

1. Improving students’ computer programming skills will improve students’ abilities to complete more complex problems and expand their mastery of the underlying concepts.
2. Improving students’ facility with LaTeX will enable them to express mathematical concepts and formulas in a standard way, which will enhance their mathematical writing.

Members of the Math Department’s Undergraduate Program Committee began developing the course in the Spring 2023 semester, with the goals of offering it on a trial basis in Fall 2024 for the first time and making it a permanent course in future semesters for all first-year math majors. We expect to have a tentative syllabus for the course completed by the beginning of the Spring 2024 semester.

As such this action is not yet complete, but we did want to write this update that in developing the course we have found that we can expand its impact. In addition to the above topics, it is natural to incorporate the university’s requirements for the course to qualify as a communication intensive course. This will involve students making presentations to the class, e.g., on their programming assignments, and writing a term paper to utilize their LaTeX skills. By adding this component to the

course students will obtain foundations for programming, presentation, and writing skills that they can carry through their math major careers.

As this course is still under development, we do not have any data to present to demonstrate its impact, but we anticipate being able to report on this aspect in future cycles.

Supporting Documentation

Measure: Applied Algebra: Comprehensive Final Exam

Summary of M443

BA - MATH			
PLO: Fundamentals of Algebraic and Geometric Methods			
Measure: Math 433 - Comprehensive Final Exam			

Academic Year	Average on Final Exam (out of 100)	% of Students who Met Standard of 65% or higher on the Final Exam	Target: 70% of the students met or exceeded the minimal standard of 65%?
2021 - 2022	96.1111111	100	Target of 70% Met
2022 - 2023	85.85714286	100	Target of 70% Met

Measure: Modern Geometry: Comprehensive Final Exam

Summary of M467

BA - MATH			
PLO: Fundamentals of Algebraic and Geometric Methods			
Measure: Math 467 - Comprehensive Final Exam			

Academic Year	Average on Final Exam (out of 100)	% of Students who Met Standard of 65% or higher on the Final Exam	Target: 70% of the students met or exceeded the minimal standard of 65%?
2020 - 2021	76.86	84.62	Target Met
2021 - 2022	84.34444444	100	Target Met
2022 - 2023	76.97619048	90.47619048	Target Met